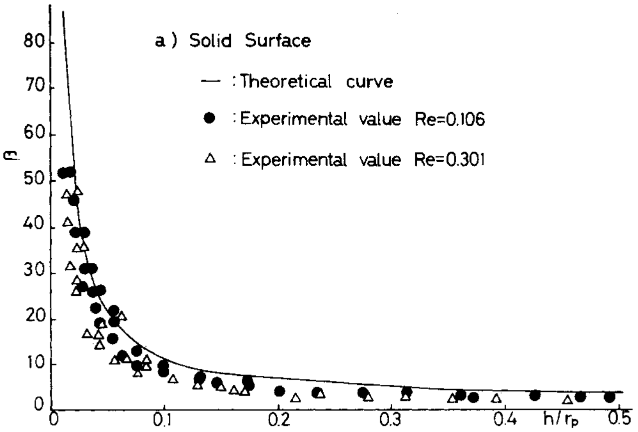

Figure 3. β **vs.** h/r_p **(solid surface).**

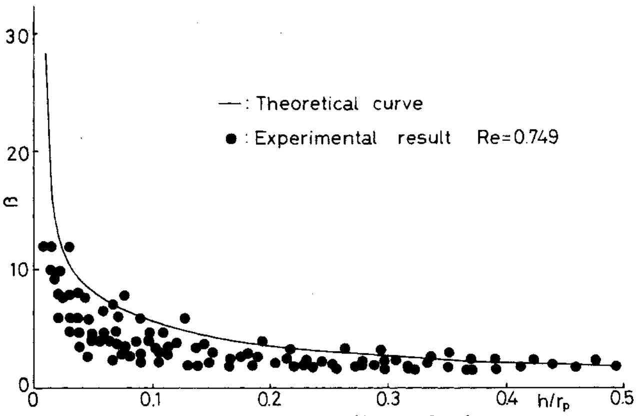

Figure 4. β **vs.** h/r_p **(free surface).**

value of u_∞ is not negligible. Its decrease in the value of u_∞ is estimated at about 32% in our condition according to Faxen's equation (1922). However, the value of u_p is also equally affected by the presence of the side walls. Therefore, the value of β is not affected much by the presence of the side walls, because the errors included in the denominator u_p and the numerator u_∞ of Eq. 6 cancel out each other. These separate effects cannot be linearly superposed (Sonshine et al., 1966). This causes some discrepancies between observed and calculated results.

It is certain that the experimental result is affected by the Reynolds number, because the increase in this number causes the experimental values to deviate considerably from the theoretical curve.

The preceding discussion on the reasons 1., 2., and 3. explains the fact that the experimental values are smaller than the theoretical ones. The spread of the experimental values is applied to the reason 4.

The experimental values in the case of the free surface spread more than those in the case of the solid surface. This is so, because it was very difficult to keep the concentration of millet jelly (i.e., the density of millet jelly) in the vicinity of the free surface homogeneous.

The comparison of Figures 3 and 4 indicates that the value of β in the case of the free surface is smaller than that in the case of the solid surface at the same h. There might be two reasons for this. First, the tangential stresses vanish on the free surface. Secondly, when the sphere approaches the vicinity of the free surface, the free surface is raised by the push of the sphere. The latter was not considered in Brenner's analysis.

NOTATION

F	= resistance force
F_w	= London-Van der Waals force
h	= minimum separation between sphere and plane surfaces
g	= gravitation acceleration
m_p	= mass of sphere
r_p	= radius of sphere
s	= distance between two marks
t	= time
u_p	= sphere velocity
u_∞	= Stokes sedimentation or rise velocity
β	= correction factor
ρ_f, ρ_p	= densities of fluid and sphere
μ	= fluid viscosity

LITERATURE CITED

Brenner, H., "The Slow Motion of a Sphere through a Viscous Fluid towards a Plane Surface," *Chem. Engng. Sci.*, **16**, 242 (1961).

Faxen, H., "Der Widerstand gegen die Bewegung einer starren Kugel in einer zähen Flüssigkeit, die zwischen zwei parallelen ebenen Wänden eingeschlossen ist," *Annalen der Physik*, **4**, Folge, 168, 89 (1922).

Mackay, G. D. M., and S. G. Mason, "Approach of a Solid Sphere to a Rigid Plane Interface," *J. Colloid Sci.*, **16**, 632 (1961).

Mackay, G. D. M., M. Suzuki, and S. G. Mason, "Approach of a Solid Sphere to a Rigid Plane Interface: Part 2," *ibid*, **18**, 103 (1963).

Sonshine, R. M., R. G. Cox, and H. Brenner, "The Stokes Translation of a Particle of Arbitrary Shape along the Axis of a Circular Cylinder Filled to a Finite Depth with Viscous Liquid: Part 1," *Appl. Sci. Res.*, **16**, 273 (1966).

Sonshine, R. M., R. G. Cox, and H. Brenner, "The Stokes Translation of a Particle of Arbitrary Shape along the Axis of a Circular Cylinder Filled to Finite Depth with Viscous Liquid: Part 2," *ibid*, **16**, 325 (1966).

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Correlation of Eddy Diffusivities for Pipe Flow

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The design of heat and mass transfer equipment requires an accurate knowledge of heat and mass transfer Nusselt numbers. These can be theoretically predicted and correlated from the turbulent transport equations, if one knows how the eddy diffusivities vary with Reynolds' number, fluid properties and the distance from the wall. Such knowledge would also be useful in the design of tubular reactors in which radial transport is important (Berker and Whitaker, 1978; Sundaram and Froment, 1979).

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Various relationships have been proposed for the diffusivities in the region near the wall and others have been suggested for the turbulent core and buffer region. However, no satisfactory single equation has yet been developed that applies to all regions and which also incorporates the effects of Reynolds' number and fluid properties. The purpose of this work was to obtain such an equation and to suggest asymptotic equations for the near-wall and core regions.

ASYMPTOTIC EQUATIONS

Near the wall, correlations of eddy diffusivities are usually of the general form:

$$\frac{\epsilon_W}{v} = \beta(y^+)^m \tag{1}$$

where β and m are constants. Although originally suggested by Murphee, Notter and Sleicher (1969) placed on firm theoretical grounds a value of $m = 3$ as $y \to 0$. This value was supported for mass transfer by the experimental data of Lin et al. (1953), and by the correlations of Hughmark (1971) for heat transfer and of Notter and Sleicher (1969, 1971) for heat and momentum transfer.

In the core region, Brinkworth and Smith (1973) fitted experimental data to a Reichardt-type distribution. This equation correctly predicts a nonzero and almost constant diffusivity at the center, in agreement with experimental data. However the Johnk and Hanratty (1962) data at various Reynolds' numbers deviated from it consistently. Therefore, in this work we have used a modified Reichardt equation with an adjustable parameter, b:

$$\frac{\epsilon_{HC}}{u_* R} = 0.063(1-r^{*2})\,(1+br^{*2}) \tag{2}$$

In many applications (Berker and Whitaker, 1978), it is not necessary to know the radial variation of the diffusivity and instead an area average diffusivity will suffice. Using the data of Johnk and Hanratty (1962), Brinkworth and Smith (1973), Smith et al. (1968) and Beckwith and Fahien (1963), values of the area-averaged thermal diffusivities were obtained in terms of the dimensionless group $<\epsilon_H>/2u_*R_H$, where R_H is the hydraulic radius. These values were found to be roughly independent of Reynolds' number. Their mean was found to be $<E_H>/2u_*R_H = 0.069$, or, in a more convenient form, $<\epsilon_H>/v = 0.0345\ Re\sqrt{f/2}$.

For comparison the average mass diffusivity obtained from the mass transfer data of Seagrave and Fahien (1961) at three values of Re was found to be about $0.078(2u^*R_H)$. The molecular Schmidt number for this system is about 3,200. Groenhof (1973) calculated an average mass diffusivity of $0.08(2u^*R_H)$ for values of Re in the range 25,800-74,900 and Sc about 750. Sherwood and Woertz (1939) measured the mass diffusivity of water vapor through gas streams in a thin duct of half width H and obtained $<\epsilon_D>/2u_*R_H = 0.567$. Thus, the mass diffusivities are within ± 20% of those for heat over a wide range of Schmidt numbers. By comparison, the widely used Prandtl-Schlichting equation gives $<\epsilon_M> = 0.0667\ (2u_*R_H)$.

GENERAL EQUATION

Of the various possibilities for nondimensionalizing ϵ in the general equation, we have selected $<\epsilon>$ since it has been correlated as part of this work and since $\epsilon/<\epsilon> \equiv \epsilon^*$ shows the radial variation relative to the average value. Converting the wall equation to ϵ^*, one uses the average value $<\epsilon_H> = 0.069\ u^*R$ and the Blasius equation $f = 0.0791/Re^{0.25}$ to obtain:

$$\epsilon^*_{WH} = 0.143\ \beta_H(Re)^{1.75}(1-r^*)^3\ (r^* \to 1) \tag{3}$$

To get the core equation, one divides Eq. 2 by its integral over the cross-section and obtains:

$$\epsilon^*_{CH} = \frac{6}{b+3}\,(1 + br^{*2})\,(1-r^{*2}) \tag{4}$$

Having asymptotic Eqs. 3 and 4, it is possible to develop an overall correlation by the use of a method developed by Churchill and Usagi (1972). In this method, the interpolation expression is:

$$\epsilon^* = [(\epsilon^*_W)^{-n} + (\epsilon^*_C)^{-n}]^{-1/n} \tag{5}$$

where the exponent n has to be determined. Since the wall equation is Reynolds' number-dependent, this method could give different values of n for each of the five Reynolds numbers at which the Johnk and Hanratty data were taken. This possible dependence of n on Re was tested by a least squares correlation (Westerberg, 1973). In the first set of runs, the five values of n were considered along with the parameters b and β_H with the result that the values of n displayed a negligible Reynolds' number dependence. Therefore, a second set of least squares correlations was carried out to determine the best fitting values of the parameters n, b, β_H and the exponent m of y^+ in the asymptotic wall equation. These results confirmed a value of $m = 3$. In addition, the best value for n was found to be nearly one,

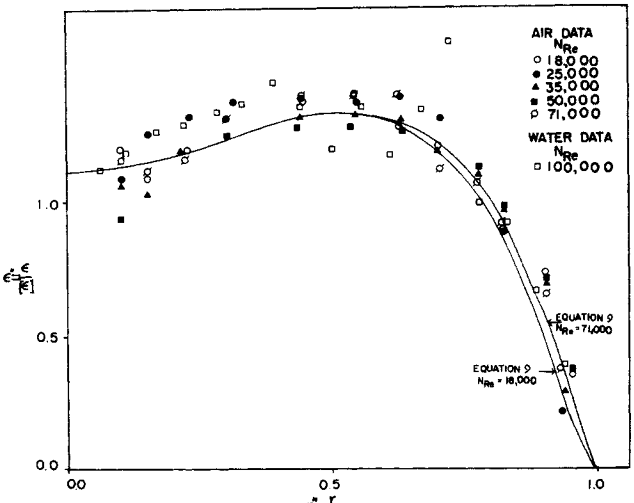

Figure 1. Dimensionless eddy diffusivity vs. dimensionless radial position.

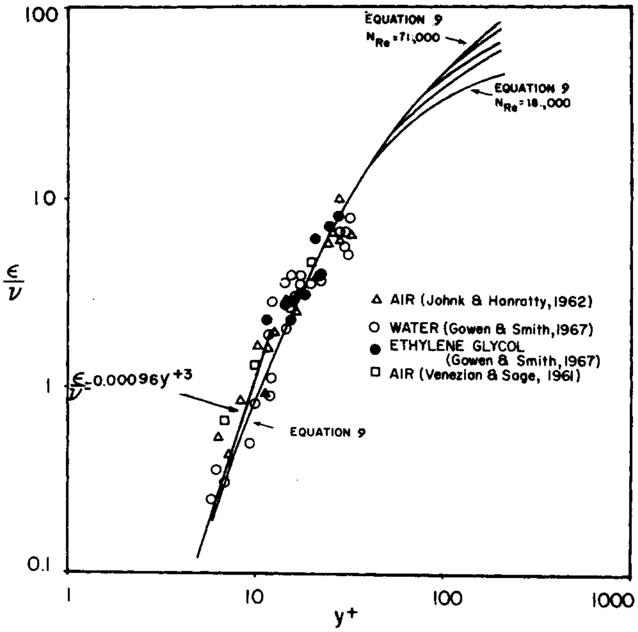

Figure 2. Diffusivities in wall and buffer regions.

with b between 2.3 and 2.4 and β_H between 0.000872 and 0.001426. Then, using $m = 3$, $n = 1$ and $\beta_H = 0.00096$ (the best fit constant for Hughmark's wall correlation), the best least square value of b was found to be 2.345.

The final correlation is then,

$$\frac{1}{\epsilon_H^*} = \frac{1}{\epsilon_{CH}^*} + \frac{1}{\epsilon_{WH}^*} \tag{6}$$

with

$$\epsilon_{WH}^* = 0.000137\, Re^{1.75}\, (1-r^*)^3 \tag{7}$$

$$\epsilon_{CH}^* = 1.123\, (1 + 2.345 r^{*2})(1-r^{*2}) \tag{8}$$

It is interesting to notice that the general expression for the diffusivity Eq. 6 can now be interpreted as a net resistance obtained as the sum of two resistances in series. Substituting Eqs. 7 and 8 into Eq. 6 gives a general equation:

$$\epsilon_H^* = \{[1.123(1 + 2.345r^{*2})(1-r^{*2})]^{-1} + [0.000137 Re^{1.75}\, (1-r^*)^3]^{-1}\}^{-1} \tag{9}$$

Plots of the above correlation together with the experimental results of Johnk and Hanratty for air and of Brinkworth and Smith for water are shown in Figure 1. The agreement is good for Re between 18,000 and 100,000. However, the data of Beckwith and Fahien for water at $Re < 18{,}000$ do not agree as well at $r^* < 0.8$.

A plot in the form ϵ_H/v vs. y^+ (Figure 2) is presented to show that the data in the transition region are well-correlated by a single correlation, thus, eliminating the need for separate expressions to describe the eddy diffusivity variation near the wall.

Although Eq. 9 was derived from asymptotic equations that fit ϵ_H data, it can be used to estimate ϵ_D and ϵ_M within 20-30% or more accuracy, if the turbulent Schmidt and Prandtl numbers are known (Hughmark, 1975; Notter and Sleicher, 1975).

The turbulent Prandtl and Schmidt numbers actually vary with radial position. At the wall, Notter and Sleicher (1975) suggest a value of the reciprocal turbulent Prandtl number of 1.3. Thus, if Eq. 7 is to be used separately to get ϵ_{WM} it should be multiplied by 1.3.

In the core, Hughmark (1975) found that a value of unity for the turbulent Prandtl number gives good predictions of the Nusselt numbers. Also, Brinkworth and Smith obtained a turbulent Prandtl number of 0.977 from the experimental data. Thus, Eq. 8 can probably be used for either heat or momentum transport. However, since Eq. 9 includes the equation for the wall region, it is strictly correct only for heat transport.

NOTATION

b	= parameter in modified Reichardt equation
H	= half width of duct
r	= radial distance from the center, cm
R	= pipe radius, m
R_H	= hydraulic radius, m
u_*	= friction velocity, m/s
$<v>$	= mean bulk flow velocity, m/s
y	= radial distance from wall, m
z	= dimensional axial variable, m

Greek Letters

ϵ	= eddy diffusivity, m²/s
v	= kinematic viscosity, m²/s

Subscripts

C	= core
D	= mass transfer
M	= momentum transfer
W	= wall condition
H	= heat transfer

Dimensionless Groups

Re	= $4R_H<v>/v$, Reynolds' number
ϵ^*	= $\epsilon/<\epsilon>$
Sc	= Schmidt number
f	= Fanning friction factor
r^*	= r/R, dimensionless radial variable
y^+	= yu_*/v dimensionless distance from pipe wall
y^*	= $1 - r^*$

LITERATURE CITED

Beckwith, W. F., and R. W. Fahien, "Determination of Turbulent Thermal Diffusivities for Flow of Liquids in Pipes," *U.S.A.E.C. Report* IS-734 (1963).

Berker, A., and S. Whitaker, "A Two-Dimensional Model of Second Order Rapid Reaction in Turbulent Reactors," *Chem. Eng. Sci.*, **33**, 889 (1978).

Brinkworth, B. J., and P. C. Smith, "Radial Distribution of Diffusivity in the Core of Turbulent Pipe Flow," *Chem. Eng. Sci.*, **28**, 1847 (1973).

Churchill, S. W., and R. Usagi, "A General Expression for the Correlation of Rates of Transfer and Other Phenomena," *AIChE J.*, **18**, 1121 (1972).

Gowen, R. A., and J. W. Smith, "The Effect of the Prandtl Number on Temperature Profiles for Heat Transfer in Turbulent Pipe Flow," *Chem. Eng. Sci.*, **22**, 1701 (1967).

Groenhof, H. C., "Eddy Diffusion in the Central Region of Turbulent Flows in Pipes and Between Parallel Plates," *Chem. Eng. Sci.*, **25**, 1005 (1970).

Hughmark, G. A., "Heat and Mass Transfer for Turbulent Pipe Flow," *AIChE J.*, **17**, 902 (1971).

Hughmark, G. A., "Additional Notes on Transfer in Turbulent Pipe Flow," *AIChE J.*, **19**, 1055 (1973).

Hughmark, G. A., "Reply to Notter and Sleicher Note," *AIChE J.*, **21**, 1701 (1975).

Johnk, R. E., and T. J. Hanratty, "Temperature Profiles for Turbulent Flow of Air in a Pipe—I. The Fully Developed Heat-Transfer Region," *Chem. Eng. Sci.*, **17**, 867 (1962).

Lin, C. S., R. W. Moulton and G. L. Putnam, "Mass Transfer Between Solid Wall and Fluid Streams: Mechanism and Eddy Distribution Relationships in Turbulent Flow," *Ind. Eng. Chem.*, **45**, 636 (1953).

Notter, R. H., and C. A. Sleicher, "The Limiting Form of the Eddy Diffusivity Close to a Wall," *AIChE J.*, **15**, 936 (1969).

Notter, R. H., and C. A. Sleicher, "The Eddy Diffusivity in the Turbulent Boundary Layer Near a Wall," *Chem. Eng. Sci.*, **26**, 161 (1971).

Notter, R. H., and C. A. Sleicher, "Diffusivity Ratios in Fully Developed Turbulent Pipe Flow," *AIChE J.*, **21**, 169 (1975).

Schlichting, H., "Boundary Layer Theory," Pergamon Press, New York (1955).

Seagrave, R. C., and R. W. Fahien, "Turbulent Mass Transfer in Liquid Streams," *U.S.A.E.C. Report* IS-419 (1961).

Sherwood, T. R., and B. B. Woertz, "Eddy Diffusion in Mass Transfer Between Phases," *Ind. Eng. Chem.*, **31**, 1034 (1939).

Smith, J. W., R. A. Gowen and B. Wasmund, "Diffusion Coefficient and Temperature Profiles for Turbulent Heat Transfer to Water in Pipes," Dallas meeting of AIChE (1968).

Sundaram, K. M., and G. F. Froment, "A Comparison of Simulation Models for Empty Tubular Reactors," *Chem. Eng. Sci.*, **34**, 117 (1979).

Venezian, E., and B. H. Sage, "Temperature Gradients in Turbulent Gas Streams: Effect of Viscous Dissipation on Evaluation of Total Conductivity," *AIChE J.*, **7**, 688 (1961).

Westerberg, A. W., *RMINSQ Program*, Personal Communication (1973).

Westerberg, A. W., *RQUADI Program*, Personal Communication (1973).

Manuscript received October 23, 1979; revision received April 21, and accepted May 6, 1980.